A STATISTICAL STUDY OF THE RELATIONSHIP BETWEEN LOCAL CHANGES IN RELATIVE VORTICITY AND CONVECTIVE WEATHER PHENOMENA

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by

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PREFACE

This paper represents a statistical evaluation of the relationship between relative vorticity and convective weather phenomena. It contains a correlation between vorticity at the 700 millibar surface and convective phenomena, and a correlation between 24 hour changes in these two variates. The data for this evaluation were obtained from 700 millibar and surface maps during the spring and early summer months of 1949 through 1951.

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The author wishes to express gratitude for the assistance of Associate Professor G. J. Haltiner, Department of Aerology, U. S. Naval Postgraduate School, Monterey, California.

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TABLE OF SYMBOLS AND ABBREVIATIONS

$\boldsymbol{\zeta}$	Vector vorticity
$\nabla \times \mathbf{V}$	Curl of velocity
ζ	Scalar relative vorticity
v	Wind speed
K_s	Curvature of a streamline
$\frac{\partial v}{\partial n}$	Wind shear along the normal to the flow
$\mathbf{n}$	Unit position vector normal to the flow
n	Element of length along $\mathbf{n}$
ζ_a	Scalar absolute vorticity
∇_H	Two dimensional grad
$\mathbf{V}$	Wind velocity
λ	Coriolis parameter
x	24-hour change in convective index
y	24-hour change in relative vorticity
B	Clear skies
BC	Partly cloudy skies
C	Overcast skies
P	Precipitation resulting from convection
R̸	Thunderstorm

I. INTRODUCTION

The inspiration for this investigation came from a paper on a similar subject by a former student of the U. S. Naval Postgraduate School, Lieutenant E. J. Writt [6] . In this paper the frequently used forecasting rule that wind flow from the south causes convergence and resulting convective phenomena was examined on a statistical basis. This was done by measuring the time rate of change of relative vorticity under nearly steady state conditions and correlating the results with the average prevailing convective phenomena. Surface maps were used in measuring relative vorticity. A linear correlation coefficient of 0.0513, based on 70 observations, was obtained.

The subject has been of interest to other investigators. Namias and Clapp [4] computed inertia trajectories on 10,000-foot maps under steady state conditions. These trajectories were such that the vertical component of the absolute vorticity of a moving parcel of air remained constant. They were determined by initial latitude, curvature, lateral shear, speed, and direction. These trajectories were not parallel to the isobars, presumably due to the modifying effects of convergence or divergence. It was found that if the isobars curved more cyclonically than the trajectories, convergence was present; if more anti-cyclonically, divergence was present.

Another paper of a similar nature was published in 1950 [2] . Relative vorticity was computed at the 700-millibar level by use of the

expression for vorticity in natural coordinates and correlated with the heights of the tops of cumuliform clouds over the ocean along the Washington-Bermuda airways. In a sample of 454 observations, a linear correlation coefficient of 0.84 was found between relative vorticity and cloud-top height. A later sample of 210 additional observations yielded a coefficient of 0.65. These results indicate a somewhat strong relation between relative vorticity and convective phenomena.

In view of the above papers, it is believed that convective phenomena can be predicted through the use of relative vorticity measurements. The goal of this paper is, therefore, to determine a simple device which the forecaster may use to improve his forecasts when convective activity is likely.

II. THE THEORY OF RELATIVE VORTICITY

Vorticity may be defined as the curl of the velocity. In rectangular coordinates,

$$\zeta = \nabla \times \mathbb{V} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\mathbb{i} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\mathbb{j} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\mathbb{K}. \qquad (1)$$

In horizontal flow, only the last term appears. However, in computing the scalar component of vector vorticity, it is more conveniently expressed in natural coordinates,

$$\zeta = v K_s - \frac{\partial v}{\partial n} \qquad (2)$$

where,

v = wind speed,

K_s = curvature of the streamlines (positive for cyclonic curvature, negative for anti-cyclonic curvature)

$\mathbb{n}$ = unit vector normal to the flow

n = element of distance along $\mathbb{n}$

$\frac{\partial v}{\partial n}$ = wind shear along the normal. Negative values are termed cyclonic shear; positive, anti-cyclonic shear.

The theorem of absolute vorticity states that the rate of change of the absolute vorticity of a moving element is proportional to its horizontal divergence [3]. It may be written as

$$\frac{d\zeta_a}{dt} = -\zeta_a \nabla_H \cdot \mathbb{V} \qquad (3)$$

where ζ_a = component of absolute vorticity and

$\nabla_H \cdot \mathbb{V}$ = horizontal divergence.

When absolute vorticity in equation (3) is replaced by its equivalent in terms of relative vorticity and the coriolis parameter, we obtain

$$\frac{d}{dt}(\zeta + \lambda) = -(\zeta + \lambda)\nabla_H \cdot \mathbb{V}. \tag{4}$$

It is apparent that a relationship exists between changes in relative vorticity and horizontal divergence.

Since it has long been established that a direct correlation exists between the horizontal convergence in the lower atmosphere and convective activity, it follows that a relationship exists between changes in relative vorticity and convection.

The purpose of this paper is to determine the nature of this relationship by extending the above theory to non-steady state conditions and computing local time change in relative vorticity rather than individual change.

III. METHOD OF COLLECTION OF DATA

Previous investigators have obtained better results through the use of upper level rather than surface charts in determining relative vorticity. Moreover, the theorem of absolute vorticity as stated by Holmboe, Forsythe, and Gustin [3] was derived on the basis of frictionless flow. Therefore, the 700 millibar level was chosen for use in computing relative vorticity. Observations were taken from the 0300Z charts covering spring and early summer of 1949 through 1951. This period was selected because it was believed that the changing situation which usually ensues at that time of the year would yield a representative spread in values of both relative vorticity and convective phenomena. Observations of convective phenomena were taken from the 0030Z surface charts covering the same period.

In computing vorticity by use of equation (2), contour lines were assumed to be streamlines. Curvature of the streamlines was determined by comparison with a series of arcs drawn on opaque paper. Reported wind speeds were recorded when available, otherwise wind speed was estimated with a geostropic wind scale. The units of curvature were nautical miles $^{-1}$, and the units of wind speed were knots. Wind shear was measured along the normal to the flow in knots per nautical mile. Hence the units of relative velocity were hours $^{-1}$.

It might be interesting to note at this point that wind shear became a contributing factor to relative vorticity only when the jet stream was in or near the area under consideration.

Convective phenomena were converted into numerical equivalents by use of a "convective index". This index consisted of weights assigned to cloud type, cloud cover, and present weather phenomena as designated by the U. S. Weather Bureau [5] . Lower cloud types 1 through 5 were assigned a value of one (1); lower cloud types 8 and 9 and middle cloud type 7 were assigned a value of two (2). The weight assigned to cloud cover corresponded to the code number representing tenths of cover [5] . Weights assigned to present weather phenomena were as follows: intermittent precipitation 3, steady precipitation 5, shower activity 7, and thunderstorm activity 9. To determine the convective index from a station model plot, the numerical value equivalent of present weather phenomenon was added to the product of the numerical values of cloud type and cloud cover.

Measurements of relative vorticity and convective index were made over the area bounded by latitudes 40 and 45 degrees north and longitudes 95 and 100 degrees west. This area was selected because it was deemed advisable to avoid one in which topography might contribute to convective activity. Values of the two variates throughout the area were averaged and recorded.

Relative vorticity measurements were recorded for all days for which the 0300Z 700 millibar chart was available. However, convective index measurements were recorded only when the area was sufficiently covered by reports of such quantity and distribution to provide a good measure of prevailing weather. No special treatment was afforded the presence of fronts or unusual phenomena.

IV. THE LINEAR CORRELATION OF SEVERAL VARIATES

A total of 286 simultaneous observations of relative vorticity and convective index were recorded. Relative vorticity ranged in value from -0.3200 to + 0.3500 and convective index from 0 to 25. A linear correlation coefficient of + 0.605 was determined for these two variates. Although this coefficient is slightly less than those determined by Crutcher, et.a., [2] , it appears somewhat significant.

Twenty four hour changes in relative vorticity were correlated with the convective index computed at the end of the period. In a sample of 266 observations in which the change in relative vorticity ranged from -0.3700 to + 0.3200, and convective index ranged from 0 to 25, a correlation coefficient of + 0.45 was obtained. This result is considered insignificant for forecasting purposes.

Twenty four hour changes in relative vorticity were then correlated with concurrent twenty four hour changes in convective index. In a sample of 264 observations in which change in relative vorticity ranged from -0.3700 to + 0.3200, and change in convective index ranged from -17 to + 18, a correlation coefficient of + 0.78 was obtained. This value appears significant, and its merits will be discussed later.

Finally, twenty four hour changes in relative vorticity were correlated with subsequent twenty four hour changes in convective index. In a sample of 251 observations, a correlation coefficient

of -0.37 was obtained. A larger coefficient from this computation would have provided a valuable forecasting tool. Possibly a shorter time change interval, such as twelve hours, would have given a larger coefficient; however, the necessary charts were not available to attempt a correlation on that basis.

In the above computations, relative vorticity data were grouped in class intervals of 5 hours^{-1}, and convective index data were grouped in class intervals of 2 units. Scatter diagrams are shown on the following pages.

Relative Vorticity (hours^{-1} x 100)													
35				1									
30								1					
25								2		1			
20	1						1	3	2		1		1
15		1		1		1	3	3	1	1	1	1	
10				5	5	5	3	6	3	4	4	1	
05	2	1	6	11	21	5	7	6	3	1			1
00	23	9	14	21	13	1	3	1	3				
-05	28	5	4	2	1	3	1	1					
-10	6	2	1	1	4								
-15	6		1		1								
-20	1		1										
-25					1								
-30			1										
	00	02	04	06	08	10	12	14	16	18	20	22	24

Convective Index

TABLE 1. SCATTER DIAGRAM OF RELATIVE VORTICITY AND CONVECTIVE INDEX.

Change in Relative Vorticity (hours^{-1} x 100)

	00	02	04	06	08	10	12	14	16	18	20	22	24
35								1					
30	1			1									
25								1	2	1			
20							1				1		
15		1					3	2		1	2		1
10	3		1		7	4	1	10	1	1	1	1	1
05	1	2	4	14	11	2	7	4	4	1	1	1	
00	17	4	7	9	12	3	2	1	4	1	1		
-05	18	4	10	6	6	2		2	1	1			
-10	12	4	3	2	1		1	1					
-15	4	3	3	2	2	2		2					
-20	3			1									
-25	4				2								
-30													
-35	1												

Convective Index

TABLE 2. SCATTER DIAGRAM OF 24-HOUR CHANGE IN RELATIVE VORTICITY AND CONVECTIVE INDEX.

Change in Relative Vorticity (hours^{-1} x 100)

	-18	-16	-14	-12	-10	-08	-06	-04	-02	00	02	04	06	08	10	12	14	16	18
35																1			
30																			
25													1	1	1			1	
20												1			1				
15											2	1	1	1	2		2		1
10							1			2	4	5	5	4	4		4	1	1
05								1	2	8	6	9	13	5	3	5			
00			2		1	1	2	9	11	26	8	5	3	1					
-05			2		4	7	8	8	8	7									
-10			1	2	4	6	7	4	3										
-15	1	1	1	5	4	2				4									
-20		1	2		1														
-25																			
-30			1																
-35					1														

Change in Convective Index

TABLE 3. SCATTER DIAGRAM OF 24-HOUR CHANGE IN RELATIVE VORTICITY AND 24-HOUR CHANGE IN CONVECTIVE INDEX DURING THE SAME PERIOD.

Change in Relative Vorticity (hours^{-1} x 100)																				
35				1																
30							1											1		
25						1	1					1								
20									1			1								
15	1		1	1			1	1	1		2		1	1			1			
10			2	2	2	1	7	3	1	5	2	2	2	1						
05		1	1	2	1	2	7	4	2	6	4	4	2	6	2	3				
00					3	1	4	3	5	7	9	4	9	6	2		1	2	1	1
-05					1		3	4	3	4	10	4	4	5	3	3	1	1		
-10										2	9	3		2	3		1		1	
-15				1	1				1	3	6	2	2	1		1		1		
-20											2					2				
-25																	1		1	
-30																	1			
-35																1				
-40												1								
	-20	-18	-16	-14	-12	-10	-08	-06	-04	-02	00	02	04	06	08	10	12	14	16	18

Change in Convective Index

TABLE 4. SCATTER DIAGRAM OF 24-HOUR CHANGE IN RELATIVE VORTICITY AND THE SUBSEQUENT 24-HOUR CHANGE IN CONVECTIVE INDEX.

V. INTERPRETATION AND EXTENSION OF RESULTS

Of the correlation coefficients determined, that of + 0.78 between the twenty four hour change in relative vorticity and the concurrent twenty four hour change in convective index is the most significant. A regression equation for these two variates is

$$x = .60\ y + .68 \qquad (5)$$

where x = 24 hour change in convective index

y = 24 hour change in relative vorticity.

In order to forecast the change in convective index from equation (5), a prognosis of the 700 millibar level is necessary. This value would then be applied to the current convective index to provide a forecast convective index. However, in most instances, such a prognostic convective index could be translated into a variety of convective phenomena. In view of this fact, a forecasting diagram such as shown in table (5) might prove useful.

This diagram was devised with the U. S. Navy verification system in mind [1] . It also requires the use of a prognostic 700 millibar chart. Forecast weather, provided it is of convective origin, is determined by entering the diagram adjacent to present weather and moving right to the change in relative vorticity as determined from current and prognostic 700 millibar charts. Such a diagram could be prepared for a given station by making a "best fit" to observed changes in relative vorticity and the weather. The two diagrams shown in tables (6) and (7) have been prepared in that manner.

The diagram shown in table (6) is the "best fit" for changes in relative vorticity at the 700 millibar level. Measurements were made at several stations taken at random in the United States during the month of April 1952. It provided 82 percent verification for 124 cases. Note that this is a rather small sample taken from several reporting stations, and it is believed that a single station over a long period of time would provide a higher percentage of verifications.

The diagram shown in table (7) is the "best fit" for changes in relative vorticity at the surface. These measurements were made at the same stations during the same period as in table (6). The resulting diagram provided 70 percent verification. At the surface the numerical value of relative vorticity is usually much smaller than at the 700 millibar level, and the resulting change in relative vorticity is correspondingly smaller.

Forecast Weather

Present Weather	-20	-15	-10	-5	0	+5	+10	+15	+20
B					B	BC	C	P	R↘
BC				B	BC	C	P	R↘	
C			B	BC	C	P	R↘		
P		B	BC	C	P	R↘			
R↘	B	BC	C	P	R↘				

Change in Relative Vorticity (hours^{-1} x 100)

TABLE 5. SKELETON FORECASTING DIAGRAM.

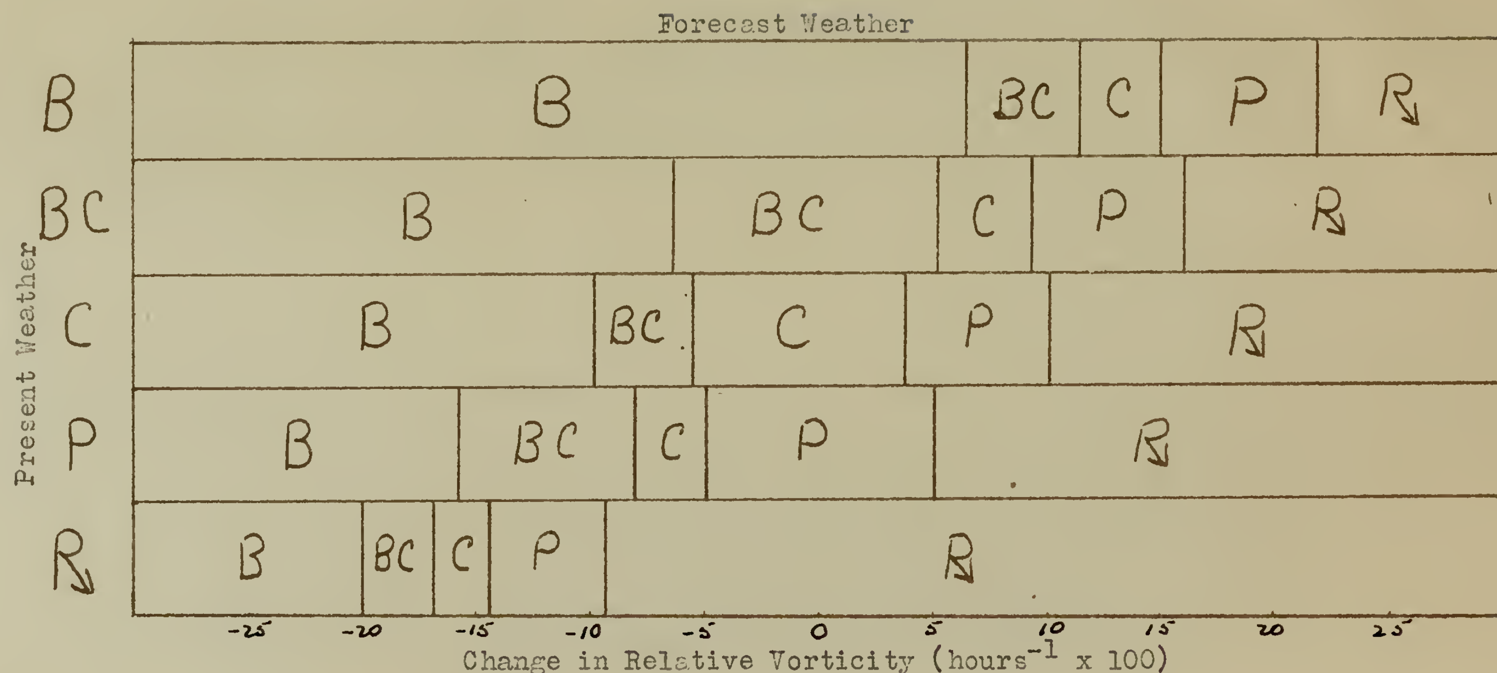

TABLE 6. "BEST FIT" FORECASTING DIAGRAM FOR 700 MILLIBAR OBSERVATIONS OF RELATIVE VORTICITY, APRIL 1952.

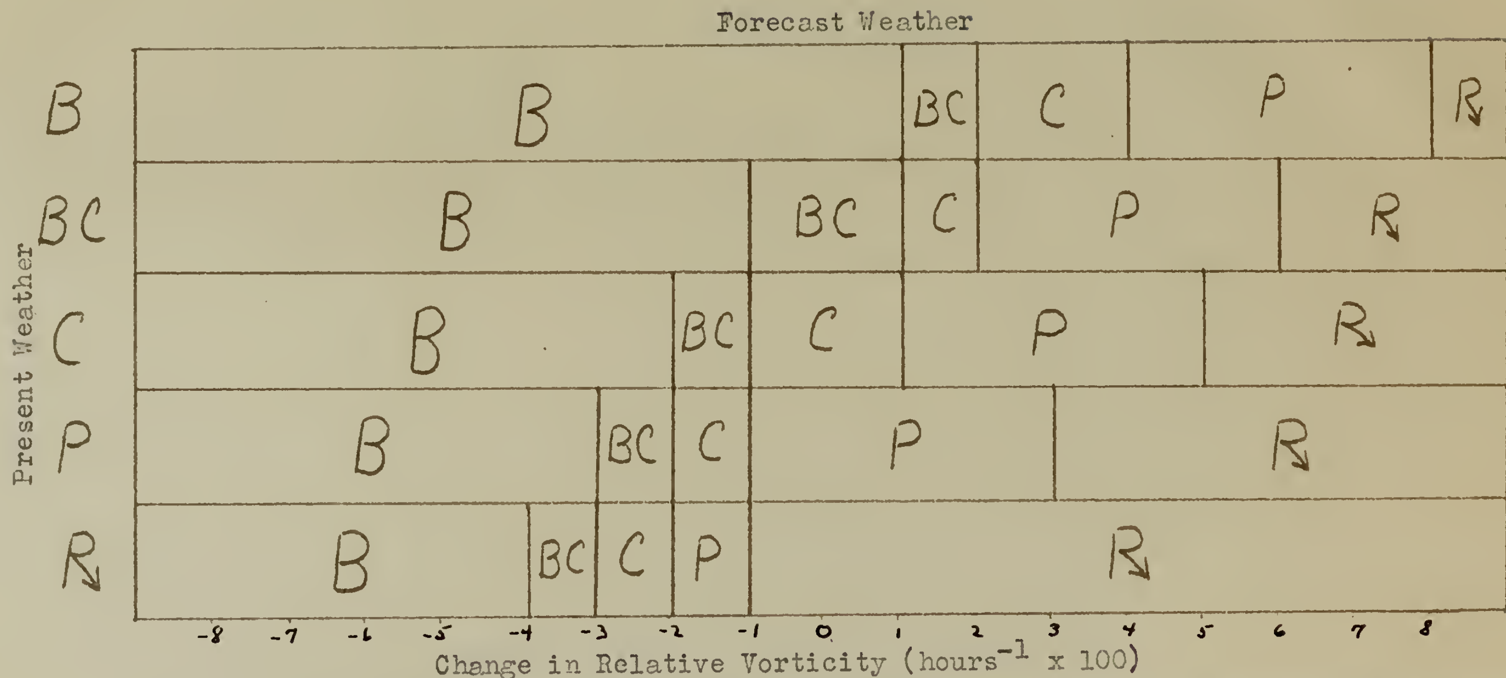

TABLE 7. "BEST FIT" FORECASTING DIAGRAM FOR SURFACE OBSERVATIONS OF RELATIVE VORTICITY, APRIL 1952.

VI. CONCLUSION

This study indicates that a relationship exists between concurrent changes in relative vorticity and convective phenomena. With the aid of fairly accurate prognostic charts, this result should be of considerable assistance to the forecaster.

No attempt has been made to correlate the moisture content of the air with convective phenomena, and no doubt such a correlation would be high. Therefore, it would appear desirable to modify the above procedure by introducing a measure of moisture content.

Probably a study of changes in these variates over a shorter interval of time, say 12 hours, would give better results. This was impractical due to the unavailability of the necessary charts.

All the above measurements were made during the spring and early summer. This seemed advisable because of the changing situation in middle latitudes at that time of the year. However, in order to be of greater value in forecasting, such a study should be carried out for the entire year, particularly the preparation of forecasting diagrams similar to those presented here.

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